# METHODS AND MEANINGS

Arc Length and Area of a Sector

The ratio of the area of a sector to the area of a circle with the same radius equals the ratio of its central angle to 360°. For example, for the sector in circle C at right, the area of the entire circle is $\pi(8)^2 = 64\pi$ square units. Since the central angle is 50°, then the area of the sector can be found with the proportional equation:
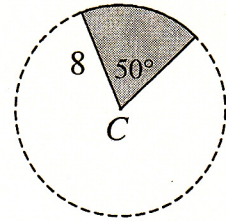
$$\frac{50°}{360°} = \frac{\text{area of sector}}{64\pi}$$

To solve, multiply both sides of the equation by 64π. Thus, the area of the sector is $\frac{50°}{360°}(64\pi) = \frac{80\pi}{9} \approx 27.93$ square units.

The length of an arc can be found using a similar process. The ratio of the length of an arc to the circumference of a circle with the same radius equals the ratio of its central angle to 360°. To find the length of $\overset{\frown}{AB}$ at right, first find the circumference of the entire circle, which is $2\pi(5) = 10\pi$ units. Then:
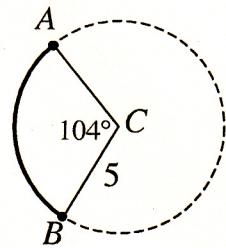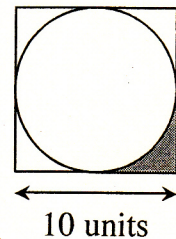
$$\frac{104°}{360°} = \frac{\text{arc length}}{10\pi}$$

Multiplying both sides of the equation by 10π, the arc length is $\frac{104°}{360°}(10\pi) = \frac{26\pi}{9} \approx 9.08$ units.

Review & Preview

8-96. The diagram at right shows a circle inscribed in a square. Find the area of the shaded region. Show all work.

10 units

8-99. An exterior angle of a regular polygon measures 18°.

a. How many sides does the polygon have?

b. If the length of a side of the polygon is 2, what is the area of the polygon?

8-100. A regular hexagon with side length 4 has the same area as a square. What is the length of the side of the square? Explain how you know.

8-101. **Multiple Choice:** Which type of quadrilateral below does not necessarily have diagonals that bisect each other?

a. square   b. rectangle   c. rhombus   d. trapezoid

8-103. A certain car's windshield wiper clears a portion of a sector as shown shaded at right. If the angle the wiper pivots during each swing is 120°, find the area of the windshield that is wiped during each swing.
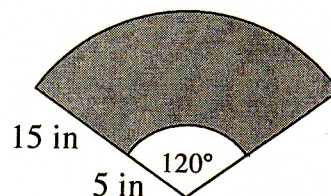
8-112. The city of Denver wants you to help build a dog park. The design of the park is a rectangle with two semicircular ends. (Note: A semicircle is half a circle.)
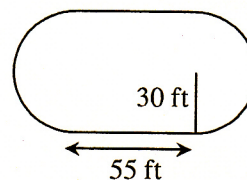
a. The entire park needs to be covered with grass. If grass is sold by the square foot, how much grass should you order?

b. The park also needs a fence for its perimeter. A sturdy chain-linked fence costs about $8 per foot. How much will a fence for the entire park cost?

8-107. Use what you know about the area and circumference of circles to answer the questions below. Show all work. Leave answers in terms of π.

a. If the radius of a circle is 14 units, what is its circumference? What is its area?

b. If a circle has diameter 10 units, what is its circumference? What is its area?

c. If a circle has circumference 100π units, what is its diameter? What is its radius?

8-109. Match each regular polygon named on the left with a statement about its qualities listed on the right.

a. regular hexagon   (1) Central angle of 36°

b. regular decagon   (2) Exterior angle measure of 90°

c. equilateral triangle   (3) Interior angle measure of 120°

d. square   (4) Exterior angle measure of 120°