Parts of a Regular Polygon

A line segment that connects the center of a regular polygon with a vertex is called a radius.

An apothem is the perpendicular line segment from

the center of a regular polygon to a side.

## The Area of a Regular Polygon

If a polygon is regular with $n$ sides, it can be subdivided into $n$ congruent isosceles triangles. One way to calculate the area of a regular polygon is to multiply the area of one isosceles triangle by $n$.

To find the area of the isosceles triangle, it is helpful to first find the measure of the polygon's central angle by dividing $360^{\circ}$ by $n$. The height of the isosceles triangle divides the top vertex angle in half.

For example, suppose you want to find the area of a regular decagon with side length 4 units. The central angle is $\frac{360^{\circ}}{10}=36^{\circ}$. Then the top angle of the shaded right triangle at right would be $36^{\circ} \div 2=18^{\circ}$.


Use right triangle trigonometry to find the measurements of the right triangle, then calculate its area. For the shaded triangle above, $\tan 18^{\circ}=\frac{4}{h}$ and $h \approx 12.311$. Use the height and the base to find the area of the isosceles triangle: $\frac{1}{2}(8)(12.311) \approx 49.242$ sq. units. Then the area of the regular decagon is approximately $10 \cdot 49.242 \approx 492.42$ sq. units. Use a similar approach if you are given a different length of the triangle.


Find the area of the shaded region for the regular pentagon at right if the length of each side of the pentagon is 10 units. Assume that point $C$ is the center of the pentagon.


8-50. Without using your calculator, find the exact values of $x$ and $y$ in each diagram below.
(a)

(b.)

c.


8-52.
Find the area of an equilateral triangle with side length 20 mm . Draw a diagram and show all work.

8-56. What is another (more descriptive) name for each polygon described below?
a. A regular polygon with an exterior angle measuring $120^{\circ}$.
b. A quadrilateral with four equal angles.
c. A polygon with an interior angle sum of $1260^{\circ}$.
d. A quadrilateral with perpendicular diagonals.

8-86. For each triangle below, find the value of $x$, if possible. Name which triangle tool you used. If the triangle cannot exist, explain why.
a.

b.

c.

d. Area of the shaded region is $96 \mathrm{un}^{2}$.


8-87. Find the measure of each interior angle of a regular 30-gon using two different methods.

8-88.
Examine the diagram at right. Assume that $\overline{B C} \cong \overline{D C}$ and $\measuredangle A \cong \measuredangle E$. Prove that $\overline{A B} \cong \overline{E D}$. Use the form of proof that you prefer (such as the flowchart or two-column proof format). Be sure to copy the diagram onto your paper and add any appropriate markings.


